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Effects of Bose-Einstein condensation on forces among bodies sitting in a boson heat bath

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We explore the consequences of Bose-Einstein condensation on two-scalar-exchange mediated forces among bodies that sit in a boson gas. We find that below the condensation temperature the range of the forces becomes infinite, while it is finite at temperatures above condensation.

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van der Waals type forces, where two photons are being exchanged [1], or the extremely feeble forces generated by the two-neutrino exchange [2,3] provide examples of forces among two static bodies in a vacuum produced by the exchange of two quanta in the t channel. Spin independent interactions arising from double (pseudo) scalar exchanges [4–6] [such as axions and/or more bizarre specimens of modern completions of the standard model (SM)] provide further examples of these so-called dispersion forces [7]. When the objects that feel such forces are placed in a heat bath at a temperature T, the forces get modified. Indeed, in the case of molecules in the relic photon background, the long-range Casimir-Polder forces among them are strongly affected for distances much larger than T^{-1} [8] and, for the two-neutrino forces, the cosmic neutrino background completely screens off the interaction at large distances [9] (again, large meaning much larger than T^{-1}).

In the present paper we shall deal with a gas of scalar bosons carrying an abelian charge and a nonzero chemical potential. As mentioned before, their double exchange between fermions has been studied in a vacuum. The case of a noncharged scalar bath in a classical Boltzmann distribution was briefly discussed in [6]. However, we are not aware of discussions on the effects resulting from placing the interacting system in a charged scalar heat bath displaying genuine quantum statistical effects such as Bose-Einstein (BE) condensation. Because in the previously reported instances, interesting effects did result, we think it is worthwhile to raise this issue here. Admittedly, light scalar bosons have a much different status than photons and neutrinos, and their nature is entirely speculative. Nonetheless, in almost any extension of the standard model, scalars are present and, furthermore, some have been suggested as candidates for dark matter so that they might be part of the cosmic relic background.

To avoid nonessential complications due to spin, we will use a simple Lagrangian that mimics the spin-independent-dispersion interactions of matter and light scalar fields. Consider the Lagrangian $\mathcal{L}_{\rm int}=g\Phi^2\varphi^2$, where Φ is a heavy scalar field of mass M and φ is a light scalar field of mass $m(m\ll M)$. Let us now put two such heavy particles Φ in a vacuum at a distance r. Their lowest-order interaction is given by the Feynman amplitude in Fig. 1. The potential is obtained from the nonrelativistic (NR) limit of this amplitude via a Fourier transformation. That is,

$$V(r)=i\int\frac{d^3\mathbf{Q}}{(2\pi)^3}e^{i\mathbf{Q}\cdot\mathbf{r}}\frac{\mathcal{T}(q\simeq(0,\mathbf{Q}))}{4M^2}\tag{1}$$

where $\mathcal{T}(q)$ is the amplitude corresponding to Fig. 1.

Taken at face value, the previous integral diverges. It has to be regulated, and the piece which leads to a long-range interaction extracted. Following [3,10], we obtain

$$V_{\rm vac}(r)=-\frac{g^2m}{64\pi^3r^2M^2}K_1(2mr)\tag{2}$$

for the potential.

For small φ mass, in the range $r\ll 1/m$, Eq. (2) shows a $1/r^3$ behavior. Beyond this range, the Bessel function gives rise to the characteristic Yukawa factor e^{-2mr}. This behavior coincides with the potential from the double exchange of (pseudo) scalars coupled to matter fermions via Yukawa couplings [5,6].

Next we introduce our system in a heat reservoir made of an ideal relativistic φ gas at temperature $T(T>m)$. We further assume that the particles in the gas carry a conserved charge corresponding to a quantum mechanical operator $\mathcal{Q}$. We may use real time finite temperature field theory [11] to calculate the effect of the heat bath on the potential between the two massive particles, taking, for the T-dependent φ-propagator,

$$D_F(k,T)=\frac{1}{k^2-m^2+i\epsilon}-2\pi i\,\delta(k^2-m^2)\times[\theta(k^0)n_+(|k^0|,T)+\theta(-k^0)n_-(|k^0|,T)],\tag{3}$$

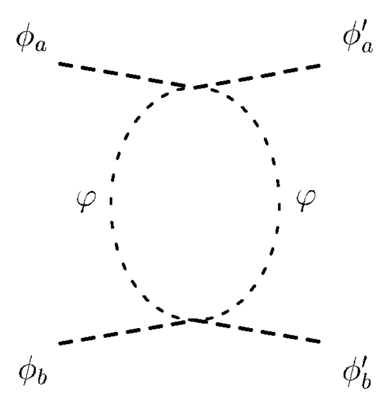

FIG. 1. Diagram giving rise to the long-range force.

where $n_{\pm}(\omega,T)=[\exp((\omega\mp\mu)/T)-1]^{-1}$ are the BE distribution functions for particles and antiparticles, respectively. μ is the chemical potential associated to the conserved charge $\mathcal{Q}$.

The amplitude of Fig. 1 now generalizes to

$$\mathcal{T}(q)=g^2\int\frac{d^4k}{(2\pi)^4}D_F(k,T)D_F(q-k,T). \tag{4}$$

This amplitude generates two distinct contributions to the potential. The first one arises from the first piece in D_F and is the vacuum potential just derived. The other corresponds to the situation where one of the scalars in the double exchange process is supplied by the thermal bath. This effect is described by the crossed terms in the amplitude involving the thermal piece of one φ propagator along with the vacuum piece of the other propagator. This thermal component of the Feynman amplitude can be written in the static limit, i.e., a momentum transfer $q\simeq(0,\mathbf{Q})$, where matter is supposed to be at rest in the frame of the heat reservoir, as

$$\mathcal{T}_T(q\simeq(0,\mathbf{Q}))=ig^2\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\sqrt{\mathbf{k}^2+m^2}}\times\frac{1}{Q^2-4\mathbf{k}^2(\hat{\mathbf{Q}}\cdot\hat{\mathbf{k}})^2}(n_++n_-), \tag{5}$$

i.e., it has been reduced to an integral over the phase space of the real particles (and antiparticles) in the heat bath.

The reservoir is thermodynamically characterized by a temperature T, a volume $\mathcal{V}$, and a fixed charge $\mathcal{Q}$ [12]. Then, the chemical potential $\mu(T)$ is determined from the relation $\mathcal{Q}=\Sigma_k(n_+-n_-)$. For a Bose-Einstein gas, the sum over states in this formula can be converted to an integral like the one in Eq. (5) as long as its temperature is above a critical temperature T_c. Below that temperature, if one makes the replacement

$$\sum_k\mapsto\mathcal{V}\int\frac{d^3\mathbf{k}}{(2\pi)^3},$$

the result is less than $\mathcal{Q}$ [12]. This is because below T_c, a large macroscopic fraction of the charge resides in the lowest energy state, and the density of states $\mathcal{V}k^2/2\pi^2$ in the continuous representation of the sum over states gives a zero weight to the zero mode. On the contrary, if the gas is above T_c then the charge is thinly distributed over the states and no individual state is populated by a macroscopic fraction of the total charge, so that by passing to the continuum, essentially, only an infinitesimally small error is incurred. Let us discuss both cases in turn. First start with the nondegenerate case, i.e., when T is above the condensation temperature T_c. In this instance, the phase space integrals in the formulas above correctly describe the physics of the problem. Therefore, we can take Eq. (5) and plug it into the expression for the potential Eq. (1). After a trivial integration over both Q and the polar angle in $\mathbf{k}$ space, we get

$$V_T(r)=-\frac{g^2}{64M^2\pi^3}\frac{1}{r^2}\int_0^\infty\frac{\mathbf{k}^2d|\mathbf{k}|}{\sqrt{\mathbf{k}^2+m^2}}\frac{\sin 2|\mathbf{k}|r}{|\mathbf{k}|}\times[n_+(\sqrt{\mathbf{k}^2+m^2})+n_-(\sqrt{\mathbf{k}^2+m^2})]. \tag{6}$$

This equation by itself is not sufficient to determine $V_T(r)$ because the functions $n_\pm$ contain the chemical potential $\mu(T)$, which has to be obtained through

$$\rho\equiv\frac{\mathcal{Q}}{\mathcal{V}}=\frac{1}{2\pi^2}\int\mathbf{k}^2d|\mathbf{k}|(n_+-n_-), \tag{7}$$

which, by the way, also determines the critical temperature T_c via the implicit equation [13]

$$\rho=\rho(T=T_c,\mu=m). \tag{8}$$

So, Eqs. (6) and (7) give the solution to our problem. We can use the high-temperature expansion of Eq. (7), derived in [13], to obtain the chemical potential as a function of T. To leading order, $\mu(T)=m(T_c/T)^2$, and we introduce it in Eq. (6) to get our potential above the condensation temperature. The total contribution to the potential, with the vacuum piece, Eq. (2), added, is finally

$$V_{T\geq T_c}^{\text{total}}=-\frac{g^2}{64M^2\pi^2}\frac{1}{r^2}T\left[e^{-2mr\sqrt{1-\xi^4}}+2\sum_{k=1}^{\infty}e^{-rT\sqrt{2(\alpha+\beta)}}\cos(rT\sqrt{2(\alpha-\beta)})\right], \tag{9}$$

with $\alpha^2\equiv(4k\pi mT_c^2/T^3)^2+(m^2(1-\xi^4)/T^2+4k^2\pi^2)^2$, $\beta\equiv4k^2\pi^2+m^2(1-\xi^4)/T^2$, and $\xi\equiv T_c/T$.

Notice in the first term of Eq. (9), the typical Yukawa damping factor cuts off the interaction at long distances compared to the Compton wavelength of φ. Furthermore, since for any k we have $T\sqrt{2(\alpha+\beta)}>2m\sqrt{1-\xi^4}$, all modes in the second term of Eq. (9) are even more suppressed at large distances.

Below T_c, a macroscopic fraction of the charge carried by particles in the reservoir piles up in the zero mode state (the condensate), and the integrals in Eqs. (6) and (7) no longer correctly describe the physical situation. Indeed, Eq. (7) gives the density of the charge in excited states ρ^*, i.e., the thermal modes [13]. For a relativistic boson gas, $\rho^*=mT^2/3$, using $\mu=m$ in this temperature regime since μ always has to be less than or equal to m, and it monotonically increases as the temperature decreases until it reaches m at T_c (and stays fixed in the macroscopical sense thereafter). Because the definition of the condensation temperature, Eq. (8), implies in this case $\rho=mT_c^2/3$, the charge density in the ground state is $\rho_0=\rho(1-(T/T_c)^2)$.

The Feynman amplitude, Eq. (5), which also involves a sum over states, should be split accordingly in two parts—the zero mode term, on the one hand, and on the other hand, the integral over thermal modes. The zero momentum mode contributes to the amplitude

$$\mathcal{T}_T(Q)|_{\mathbf{k}=0} = \frac{ig^2}{mQ^2}\frac{1}{\mathcal{V}}(n_+ + n_-)|_{\mathbf{k}=0}, \tag{10}$$

where the distribution function factor can be rewritten as

$$\frac{1}{\mathcal{V}}(n_+ + n_-)|_{\mathbf{k}=0} = \frac{\mathcal{Q}}{\mathcal{V}}(1-(T/T_c)^2) + \frac{1}{\mathcal{V}}\frac{2}{e^{2m/T}-1} \tag{11}$$

since $\mu = m - \mathcal{O}(T/\mathcal{Q})$.

As long as the net charge $\mathcal{Q}$ is a macroscopically large number many orders of magnitude larger than T/m, this factor essentially coincides with the condensate contribution to the density ρ_0. One may gain intuition on how a charge is distributed among states by making a few numerical exercises with our formulas. By way of example, we take a fiducial volume of 10 m^3 filled with 400 units of charge per cubic cm (i.e., numerically equal to the photons in the microwave background radiation). Then, for $m = 10^{-6}$ eV, 4×10^{11} particles and 15 antiparticles populate the ground state, while 4.6×10^8 particles and 4.2×10^8 antiparticles fill the excited states at $T = 0.01\ T_c = 3\times 10^{-4}$ eV. Clearly, the statement following Eq. (11) is correct, and it allows us to calculate $\mathcal{T}(Q)|_{k=0}$ in terms of the fixed quantity ρ:

$$\mathcal{T}_T(Q)|_{k=0} = \frac{ig^2}{3Q^2}(T_c^2 - T^2). \tag{12}$$

The Fourier transform of this equation gives the contribution of the condensate to the potential. It is

$$V_0(r) = -\frac{g^2}{48M^2\pi}\frac{1}{r}(T_c^2 - T^2). \tag{13}$$

The thermal contribution (i.e., from the excited states) is just Eq. (6) with the chemical potential held fixed at the constant value $\mu = m$. We find an expression that coincides with Eq. (9) at $T = T_c$, and it contains an infinite sum as well. As before, all terms in the infinite sum decay faster than e^{-2mr} for $T > m$. Therefore, for distances much larger than the Compton wavelength of φ, i.e., $r \gg m^{-1}$, and hence, $r \gg T_c{}^{-1}$, the main contribution to $V^{\text{total}}_{T\leqslant T_c}$ comes from Eq. (13):

$$V^{\text{total}}_{T\leqslant T_c} \simeq -\frac{g^2}{48M^2\pi}\frac{T_c^2}{r}[1 + \mathcal{O}(T^2/T_c^2, T/rT_c^2)]. \tag{14}$$

Inspection of these results immediately leads us to realize an important consequence of Bose-Einstein condensation. Namely, at low temperature (i.e., below T_c), the force, that was finite ranged at high temperature (i.e., above T_c), becomes infinite ranged. This comes about because the medium absorbs and restores the three-momentum in the scattering process, so that the four-momentum squared of the other φ quantum exchanged in the t channel can reach the mass shell in the physical region of the scattering process. For the $\mathbf{k} = 0$ mode in the bath, in particular, the propagator of the second particle [see Eq. (5)] takes the form of the Coulomb propagator, and becomes singular at the edge of the $\mathbf{Q}$-integration region, exactly as in the Coulomb case. For

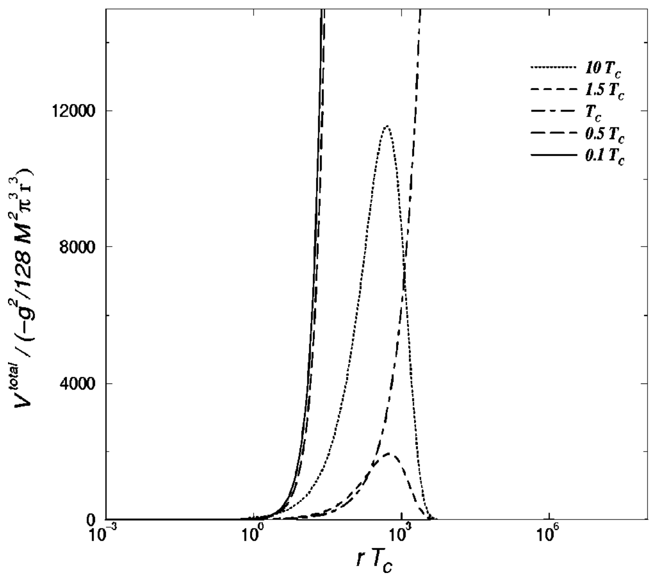

FIG. 2. Total potential, divided by $-g^2/128M^2\pi^3r^3$, for a relativistic bose gas made of particles of $m\sim 10^{-6}$ eV and having a density $\rho\sim 40\ \text{cm}^{-3}$. Behavior above and below T_c is shown. Note the Yukawa exponential damping for $T > T_c$, starting at $r \sim 1/m \Rightarrow rT_c \sim 10^3$.

$T > T_c$, this infinite wavelength mode has zero measure, and it does not contribute to the potential. However, in the condensed phase, the infinite-range potential arises as a collective phenomenon, essentially because all of the charge piles up in the ground state.

What we would like to do now is to graphically show the transition of the potential as we vary the temperature from $T > T_c$ to $T < T_c$, by numerically evaluating the infinite sums required. Figure 2 displays our results.

Let us briefly summarize our findings. Light scalars are basic ingredients of many completions of the standard model. They may carry a new conserved quantum number. If ordinary matter is neutral with respect to this new charge, then these scalars should couple to ordinary matter in pairs. But double (pseudo) scalar exchange generates long range spin independent forces among bulk matter, exactly as two-neutrino exchanges and two-photon exchanges (van der Waals forces) do. All these dispersion forces are modified when matter is introduced in a heat bath. The present paper presents an investigation of the effects of a relativistic ideal Bose gas on potentials generated by a two-scalar exchange. An example for such a heat bath could be provided by hot dark matter (i.e., relativistic at decoupling) made of hypothetic relic light scalars. For this purpose we use a very simple model for matter-scalar interactions that correctly reproduce the large r behavior of two-(pseudo)scalar exchange potentials. We do not want to commit ourselves to any specific extension of the SM, and the phenomena produced by Bose-Einstein condensation is totally independent of the form of the interaction chosen. What we find is a very dramatic effect: below the critical temperature, the finite-range force that we had above this temperature becomes an infinite-range force. The phenomenon arises as a combination of kinematics (three-momentum exchange of the matter system with the medium) and the collective effect of condensation of the charge. In the case studied in this paper, a potential of the form $\sim\exp(-2mr)/r^2$ at $T>T_c$ converts to a $\sim 1/r$ potential at $T<T_c$. Should hot relic scalars populate our Universe with a present density such that their temperature is below the threshold for Bose-Einstein condensation, then the effect described above would provide an excellent opportunity for experiments searching for forces weaker than gravity [14] since in this case, no exponential decay with distance occurs and, furthermore, a milder power law falloff with distance ensues.

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